# Interface Jump Conditions: Eliminating Spurious Velocities in Free Surface Flows

**Vuko Vukčević[1] and Hrvoje Jasak[1,2]**

[1]Faculty of Mechanical Engineering and Naval Architecture, Croatia,
[2]Wikki Ltd, United Kingdom

[1]vuko.vukcevic@fsb.hr, [2]h.jasak@wikki.co.uk, hrvoje.jasak@fsb.hr

**Abstract**

*This work presents a mixture two-phase flow model which eliminates spurious air velocities near the interface. The mathematical model is based on jump conditions at the interface, and is implemented in foam–extend. The model is validated on three test cases.*

## 1. Introduction

Neglecting surface tension and viscosity, hydrostatic momentum equation reads:

$$\frac{\partial(\rho \mathbf{u})}{\partial t} = -\nabla p_d - \mathbf{g}\bullet\mathbf{x}\nabla\rho = \mathbf{S_u}\,.$$

Source term $\mathbf{S_u}$ represents the imbalance between the dynamic pressure gradient and density gradient present at the interface. Hence, $p_d - \rho$ (or $\alpha$) coupling is resolved within the momentum equation: their imbalance may cause the lighter fluid to spuriously accelerate.
We propose an approach where $p_d - \rho$ coupling is resolved in the pressure equation, using second order accurate discretisation of pressure jump conditions near the interface.

## 2. Mathematical Model

- Mixture volumetric continuity equation:

$$\nabla\bullet\mathbf{u} = 0\,,$$

- Mixture formulation of the momentum equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla\bullet(\mathbf{u}\mathbf{u}) - \nabla\bullet(\nu_e \nabla \mathbf{u}) = -\frac{1}{\rho}\nabla p_d\,,$$

- Volume-of-Fluid phase continuity equation:

$$\frac{\partial \alpha}{\partial t} + \nabla\bullet(\mathbf{u}\alpha) + \nabla\bullet(\mathbf{u}_r \alpha(1-\alpha)) = 0\,,$$

- Interface pressure jump conditions:

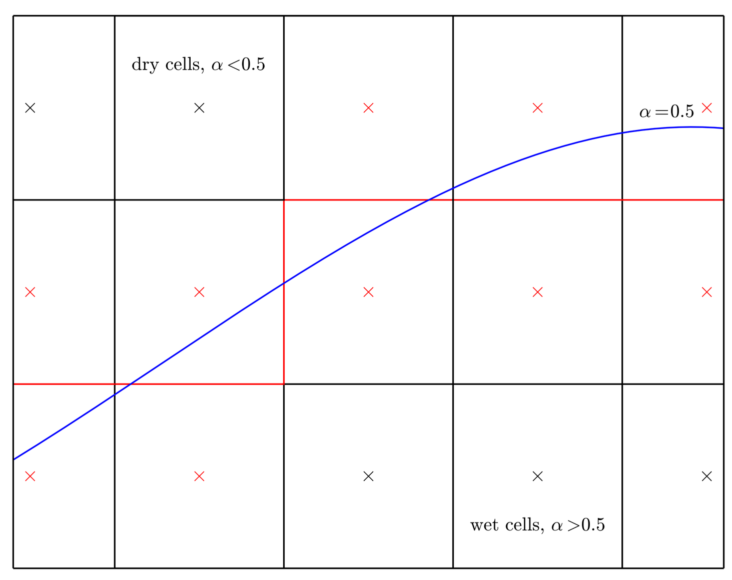
$$[p_d] = -[\rho]\mathbf{g}\bullet\mathbf{x} \quad \text{and} \quad \left[\frac{1}{\rho}\nabla p_d\right] = 0\,.$$

where $[\cdot]$ denotes the jump across the interface. Tangential stress balance at the interface is approximated by smeared effective viscosity: approach is exact for inviscid flows and justified for large–scale flows in naval hydrodynamics.

## 3. Numerical Procedure

Second order accurate discretisation of the pressure jump conditions yields corrected interpolation schemes for the pressure (gradient, Laplacian). Corrected interpolation schemes are applied only for interface faces, see Fig. 1. $\alpha$ is used only to calculate second order approximate of interface location and effective blended dynamic viscosity. Only individual (constant) densities of two fluids are used, i.e. cell averaged $\rho = \alpha\rho_1 + (1-\alpha)\rho_2$ is **never used** within the solution algorithm.

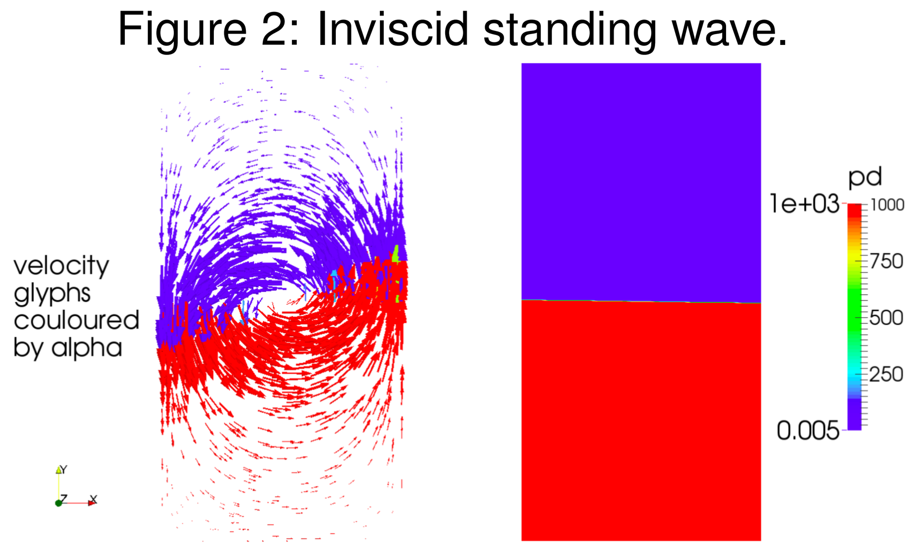
Figure 1: Interface faces (marked red).




## 4. Inviscid Standing Wave

Small amplitude, inviscid standing wave is simulated and compared to analytical solution. Fig. 2 shows the velocity field in water and air, and dynamic pressure with a jump across the interface. Relative error for wave height and zero crossing period is lower than 1%.

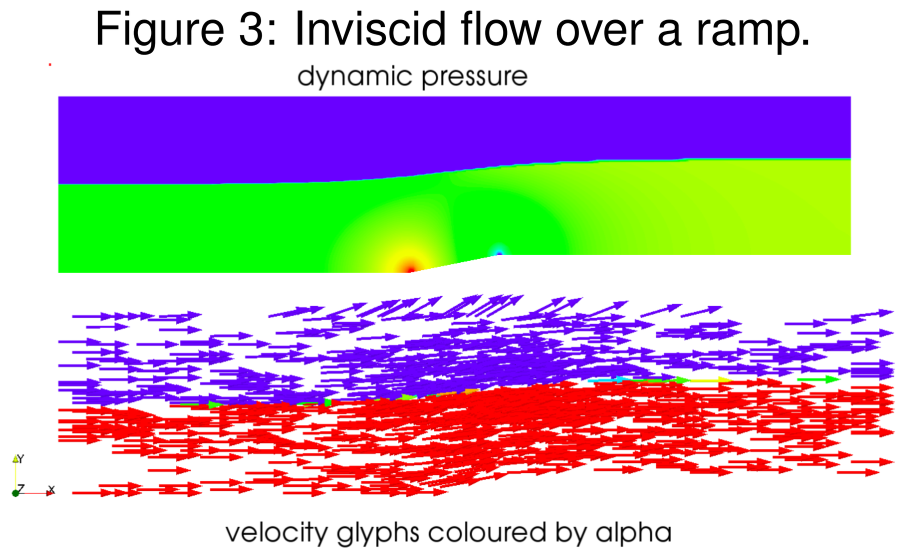
Figure 2: Inviscid standing wave.




## 5. Inviscid Free Surface Flow Over a Ramp

Inviscid free surface flow over a ramp is simulated and compared with the analytical solution. Fig. 3 shows velocity and dynamic pressure fields at steady state. Relative error for water height at the outlet is 0.34%.

Figure 3: Inviscid flow over a ramp.




## 6. MOERI Container Ship - KCS

Steady resistance simulations for the KCS hull with dynamic sinkage and trim is carried out for 6 Froude numbers, using the mesh with 950 000 cells. Results are compared with experimental data (Tokyo 2015 CFD Workshop). Fig. 4 shows the velocity field at the symmetry plane. The largest velocity is 2.88 m/s, compared to forward speed of 2.196 m/s. Fig. 5, 6 and 7 show the drag coefficients, sinkage and trim comparison.

Figure 4: KCS at design Froude number.

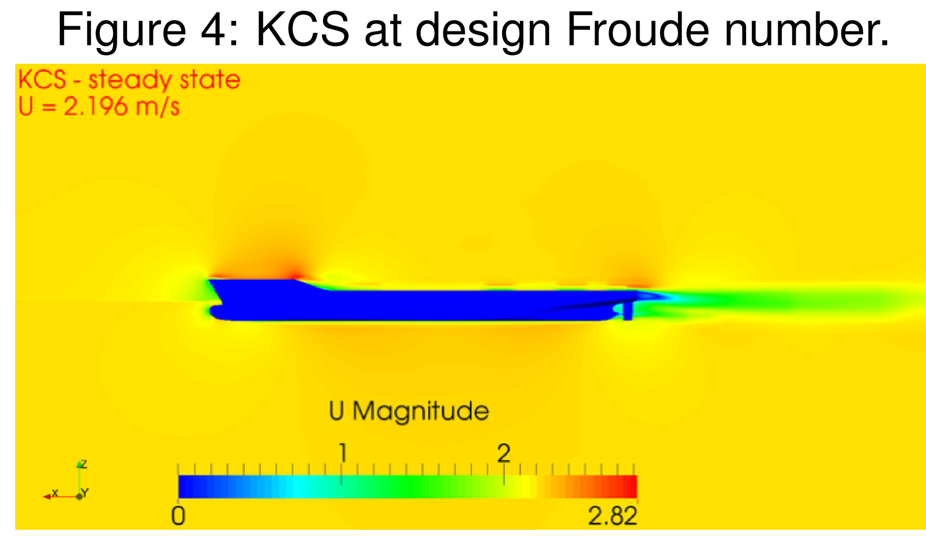



Figure 5: Drag force coefficient for different Froude numbers.

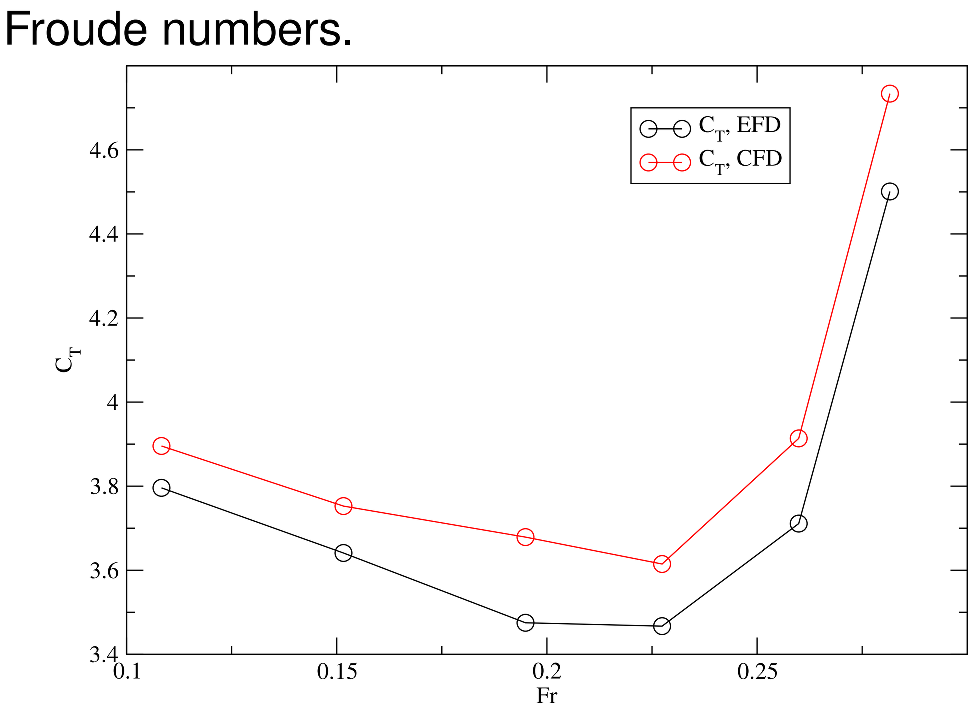



Figure 6: Sinkage for different Froude numbers.

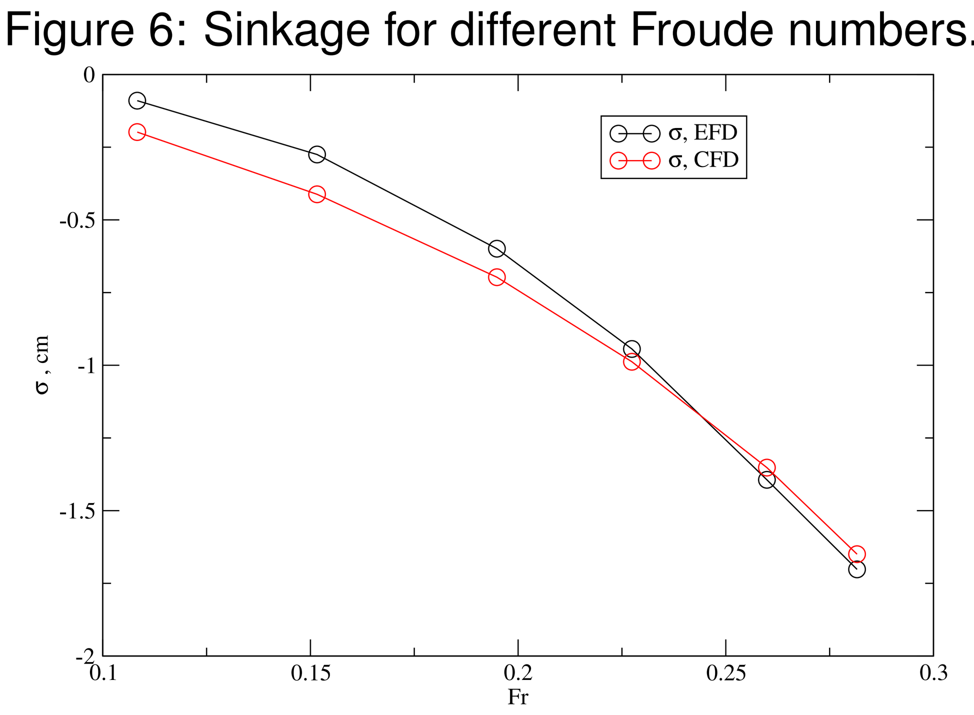
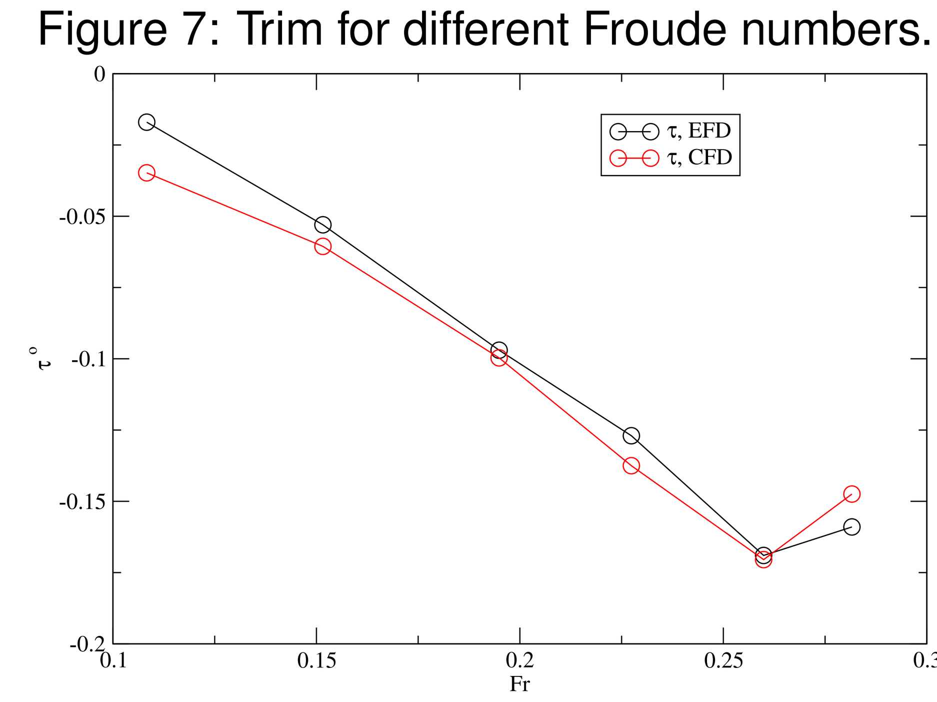



Figure 7: Trim for different Froude numbers.




## 7. Conclusion

Implementation of the interface jump conditions successfully resolved spurious air velocities near a free surface. Comparison with analytical solutions demonstrates the validity of the method for inviscid flows. KCS simulations validate the presented method for steady resistance problems, showing good agreement across a range of Froude numbers. Largest discrepancy is observed for sinkage and trim at low Froude numbers because of the insufficient mesh resolution compared to small ship motion.

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